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AUTHOR McNeil, Keith; And Others
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ABSTRACT

The utility of a non-linear transformation of the criterion is established. The Pythagorean Theorem is used as the example to demonstrate the point. The functional relationships may be such (as in the Pythagorean Theorem) that an R-squared of 1.00 cannot be found without making a non-linear transformation of the criterion. The goal of predictability (R-squared=1.00) thus may not be reached without making a non-linear transformation of the criterion. (Author/JKS)

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NON-LINEAR TRANSFORMATION OF THE CRITERION

Keith McNeil — NTS Research Corporation
Joe Evans — Evaluation Technology
Judy McNeil — NTS Research Corporation

The utility of a non-linear transformation of the criterion is established. A well-known law from a field other than education is used as the example to demonstrate the point. The functional relationships may be such (as in the Pythagorean Theorem) that an R^2 of 1.00 cannot be found without making a non-linear transformation of the criterion. The goal of predictability (R^2=1.00) thus may not be reached without making a non-linear transformation of the criterion.

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NON-LINEAR TRANSFORMATION OF THE CRITERION

There seems to us to be little justification for the present over-dependence in the behavioral sciences on the investigation of linear relationships. Since many of the simplest functional relationships in the physical sciences have been found to be non-linear or interactive, we find it interesting that few non linear relationships have been established in the behavioral sciences, especially since most behavioral scientists would maintain that human behavior is no less complicated than physical behavior. We believe there are a number of historical reasons for the emphasis on linear relationships and these will be reviewed later.

There are some historical precedents, though, which provide excellent examples of the value of considering non-linear relationships. Newton's development of the law of gravity ($d = \frac{1}{2}gt^2$) within the framework of multiple linear regression has already been discussed elsewhere (McNeil, 1970; and reprinted in McNeil, Kelly and McNeil, 1975). Newton's functional relationship relates the criterion (distance) to only one predictor variable, and that one variable is an interaction between a linear term and a non-linear term (gravity and the square of time). No linear interactions, no linear terms, and no "constants" were needed. We have hopes that the Newton discussion has influenced a number of behavioral scientists to investigate non-linear transformations of predictor variables. We turn now to discuss non-linear transformations of criterion variables.

Pythagoras Had a Reason for Considering a Non-Linear Transformation of the Criterion

Pythagoras was attempting to determine the length of the longest side of a right triangle from the lengths of the other two sides. Using regression models with linear terms was leading to only moderate success ($R^2 = .80$):

Model 1: $C_1 = a_0U + a_1A_1 + a_2B_1 + E_1$

Where: C_1 = length of longest side of right triangle;

U = 1 for all triangles;

A_1 = length of one of the shorter sides;

B_1 = length of the other shorter side; and

a_0, a_1, and a_2 are least squares weighting coefficients calculated so as to minimize the error in E_1.

After reading the paper describing Newton's success, Pythagoras considered some non-linear transformations:

Model 2: $C_1 = a_0U + a_1A_1 + a_2B_1 + a_3A_1^2 + a_4B_1^2 + E_2$

Where: A_1^2 = the square of the value in A_1; and

B_1^2 = the square of the value in B_1.

When using the above model, the R^2 increased to only .83, a non-significant increase. Pythagoras got frantic and gave the prediction terms the fifth degree:

Model 3: $C_1 = a_0U + a_1A_1 + a_2B_1 + a_3A_1^2 + a_4B_1^2 + a_5A_1^3 + a_6B_1^3 + a_7A_1^4 + a_8B_1^4 + a_9A_1^5 + a_{10}B_1^5 + E_3$

Alas, the increase in predictability was still only minimal: $R^2 = .84$.

A flash of insight (akin to a falling apple) led Pythagoras to transform the criterion, ending up with the following model:

Model 4: $C_1^2 = a_0U + a_1A_1 + a_2B_1 + a_3A_1^2 + a_4B_1^2 + a_5A_1^3 + a_6B_1^3 + a_7A_1^4 + a_8B_1^4 + a_9A_1^5 + a_{10}B1^5 + E_3 + E_4$

(Actually, the criterion variable was squared accidentally when Pythagoras was hastily transforming the predictor variables. Hence another in a long history of serendipitous findings.) As with most flashes of insight, the results led to an R^2 of .999 ... Pythagoras was delighted in finally reaching the first Goal of Research--Predictability. Pythagoras realized,

of course, that the scientific community was not really interested in accounting for variance in C_1^2; their interest was in C_1. But Pythagoras realized that this new model would allow the indirect prediction of C_1 and would do it better than previous models.

Pythagoras also realized that there were other Goals of Research: Parsimony, Replicability, Validity Generalization, Control, and Explanation. (See McNeil, et al, 1975 for discussion of these Goals within the regression framework.) In striving for a more Parsimonious model, Pythagoras considered some restricted cases of Model 4, one of the more parsimonious being:

Model 5: $C_1^2 = a_0U + a_1A_1 + a_2B_1 + a_3A_1^2 + a_4B_1^2 + E_5$

Model 5 continued to yield an R^2 of .999... but contained only <u>five</u> pieces of information. Eliminating the second degree terms from Model 5 reduced the R^2 to .81, a significant and non-acceptable reduction. That is, Pythagoras preferred the less parsimonious model (Model 5) because of its higher degree of predictability.

Fortunately for Pythagoras, a maverick statistician was available for consultation and recommended eliminating either one or both linear terms from Model 5. Most statisticians at that time felt that all lower power terms belonged in a model. The maverick statistician, as well as Pythagoras, realized that the only terms which belonged in a model were the ones which were useful for prediction. Restricting the linear terms from Model 5 yields:

Model 6: $C_1^2 = a_0U + a_3A_1^2 + a_4B_1^2 + E_6$

Model 6 yielded an R^2 of .999... and the numerical values for the weighting coefficients were $a_0 = 0$; $a_1 = 1$; and $a_2 = 1$. Thus the unit vector could also be eliminated from the model, leaving only two pieces of information in

the model. Any further restrictions (eliminating either A_1^2 or B_1^2) produced an R^2 too low to be acceptable. Thus, Model 7 was considered to be a better model because it met the Goals of Research of Predictability and Parsimony simultaneously:

Model C: $C_1^2 = a_3A_1^2 + a_4B_1^2 + E_7$

Pythagoras was concerned about Replicability, because Model 7 was obtained through data snooping and may well have been applicable only to the original set of data. Indeed, the high degree of predictbability (including the values of the weighting coefficients) of Model 7 held up under other sets of data, thus meeting the Goal of Replicability.

The Goal of Validity Generalization did not prove to be of much concern to Pythagoras. Model 7 held up (continued to yield an R^2 of .999...) under a number of diverse conditions--such as color of right triangle, medium on which the right triangle was drawn, and altitude at which the right triangle was measured. (Newton's models did not hold up under varying altitudes and thus had to be modified before the model could validly be generalized across various altitudes.) The fact that Pythagoras' model was applicable to various kinds or right triangles meant that the model was not limited in scope, but was applicable to situations which were of interest to the scientific community at that time.

The Goal of Control was investigated by drawing the two sides which were at 90° to each other and then predicting the length of the hypotenuse. Additionally, varying the length of one of the two 90° sides had predictable effects on the hypotenuse. These studies confirmed that the length of the hypotenuse could be controlled (in a predictable fashion) by manipulation of either or both of the two 90° sides.

Linear and Non-Linear Relationships

The majority of the literature in any field has consisted of linear relationships. Hays (1963) indicates three major reasons why reasearchers have focused of linear relationships, none of which seem to be persuasive reasons to continue the practice.

First, "as a practical first approximation to a complicated relationship, a linear function rule often serves quite well" (Hayes, 1963, p. 565). This may be true in terms of a measure approximating a construct. But in relating one variable to another, many researchers have finally determined that either a U-shaped or an inverted-U-shaped functional relationship exists between two variables. In such an instance, the bivariate correlation of both the linear component with the criterion and second degree component with the criterion will be close to zero; and if one were using a stepwise regression program neither would end up in the "best model". The "best model" simultaneously requires both the linear and second degree components. Hays (1963) goes on to say that, "the needs of practical prediction may be met quite well by a simple linear regression equation". The point that we would like to make is that settling for a moderate degree of predictability may no longer be an appropriate stance to take. In this day of abundant literature, ease in communication, funding for wide-scale studies, and availability of computer processing, one should attempt to achieve a fairly high degree of predictability--especially when predictability might be achieved by better use of data already collected, not at the expense of extra data collection.

As a second reason for the past focus on linear relationsips, Hays reminds us of the tie between correlation and multivariate normal assumptions. If one's data is normally distributed, then "the absence of a linear relation is the absence of any systematic relation" (Hays, 1963, p. 565). We only

need to point out that many interesting criteria and predictors are not normally distributed. In addition, a "normally distributed" criterion could be highly predicted by a set of non-normally-distributed predictors.

Hays' third point is that many measurement and statistical techniques have been developed on the assumption of linear relationships. We would like to point out that the vast majority of computer programs deal only with linear relationships, or at least make it difficult to consider non-linear relationships. On the other hand, most computer installations have a linear regression program which can investigate non-linear relationships. We find hope in Hays' statement that, "current practice and practical necessity should not be confused with the way nature or man behaves" (1963, p. 565). We have been known to express this same notion a little bit more strongly by pointing out that God did not make the world linear--all that one has to do to verify this point is to look closely at the horizon.

Why Investigate Non-Linear Relationships?

There are two major situations wherein one would investigate non-linear relationships. As discussed in more detail in McNeil, Kelly and McNeil (1975), one situation is when the functional relationship is indeed non-linear. The second situation is when the measure of the construct does not map the construct, and some rescaling of the measure is necessary.

Newton's law of gravity ($d = \frac{1}{2}gt^2$) is an example of a non-linear functional relationship. Any instance wherein raw scores are transformed to standard scores, or any type of scale, would be an example of a rescaling of the measure. Multiple linear regression rescaling applications in the areas of 1) difficulty of items, 2) ceiling effect, 3) guessing effect, and 4) conceptual rescaling are given in McNeil et.al. (1975, pp. 292-300).

The Pythagoras example required a non-linear transformation of the

criterion in order to ascertain the functional relationship. Usually, one is interested in accounting for variance in the criterion in order to understand why subjects vary on the criterion. Once Pythagoras made a non-linear transformation on the criterion, the interest was on accounting for variance in the transformed criterion for the sole purpose of understanding why subjects vary on the untransformed criterion.

Conclusion

Pythagoras was not interested in accounting for the variance in the non-linear criterion per se. Instead, Pythagoras was interested in determining the functional relationship between the length of the hypotenuse of a right triangle and the length of the other two sides. The use of a transformed criterion led to the solution of the problem. Indeed, Pythagoras could not have obtained an R^2 of 1.00 by considering only the non-transformed criterion. That is, the hypotenuse is only determinable from the two sides of the triangle by the indirect route of the square of the hypotenuse.

Pythagoras was interested in determining the precise relationship between the hypotenuse and the other two sides of a right triangle. He was correct in measuring the two sides and using them as predictors of the hypotenuse. Although the initial results yielded an R^2 or .80, this R^2 was not sufficient for purposes of practical predictability, such as building bridge supports with right triangles. Pythagoras would have been wrong in assuming that the small amount of error existing in the initial results was due to the existance of some unmeasured variable(s) or to error of measurement. The initial results were due to the fact that the original measurements (both predictors and criterion) were not organized into the proper functional relationship.

References

Hays, W.L. Statistics for Psychologists. New York: Holt, Rinehart, and Winston, 1963.

McNeil, K.A. Meeting the goals of research with muliple linear regression. Multivariate Behavioral Research, 1970, 5, 375-386.

McNeil, K.A., Kelly, F.J., and McNeil, J.T. Testing Research Hypotheses Using Multivariate Linear Regression. Carbondale, Illinois: Southern Illinois University Press, 1975.